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Global injected power statistics in a turbulent system: degrees of freedom and aspect ratio effect

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Abstract. The properties of the global energy injection rate of a closed turbulent flow produced between two counter-rotating disks are studied. The statistics of this global quantity are measured when the aspect ratio (defined as the ratio of the disk diameter to the separating distance between both disks) is modified. It is shown that the non-Gaussian statistics obtained at a low aspect ratio becomes Gaussian for a large aspect ratio. This effect is accompanied with a large decrease of the fluctuation rate. These results indicate the total energy injection rate to obey the central limit theorem. It is also shown that the number of degrees of freedom of total injection rate may be related to the number of independent large scale structures present in the flow.

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Rapid Note

1 Introduction

Global quantities can generally be seen as a sum of independent variables (the degrees of freedom) having the same statistics. Hence, for a large number of degrees of freedom, the statistics of the global variable should tend to a Gaussian statistic (central limit theorem). Actually, if N is the number of degrees of freedom for a global quantity $G(t)$ taken as the binomial law, one should have the behavior $\langle G\rangle \sim N$ for the mean, $G_{rms} \sim \sqrt{N}$ for the fluctuations and $\frac{G_{rms}}{\langle G\rangle} \sim \frac{1}{\sqrt{N}}$, for the rate of fluctuations.

For the last ten years, numerical and experimental works [1–10] have studied the global power injected (i.e. the global energy injection rate) in a turbulent flow. All of these works characterized the probability distribution function (PDF) of the injected power. The sign of the PDF's asymmetry is not universal and depends on the way the turbulence forcing is performed. Particularly, in [9] it is demonstrated that the negative skewness of the power fluctuations observed for a constant velocity forcing is equivalent to a positive skewness at constant force forcing.

In reference [3], the effect of the Reynolds number on the injected power statistics was investigated. The fluctuation rate (defined as the ratio of root mean square to the mean value) was found to decrease as the Reynolds number Re was increased showing somehow, that the number of degrees of freedom increases with Re. On the contrary, in an identical turbulent flow geometry, [8] obtained a fluctuation rate independent on the Reynolds number (another experiment [4] also leads to same conclusion). Moreover, the PDFs statistics becomes more and more asymmetric as the Reynolds number is increased. Hence, if the central limit theorem is applicable to turbulent flows, the results obtained in [8] indicate that the number of degrees of freedom is not a function of the Reynolds number. This observation is in disagreement with ideas ([3,12]) that relate the degrees of freedom of the injected power to some turbulence small scales based on the Reynolds number as either the Taylor micro-scale or the Kolmogorov dissipative scale. The discrepancy between the results of reference [3] and [8] are discussed in [8] and can be due to the way the mechanical power is estimated from the electrical consumed power. The measurements of [8] have been confirmed by numerical computations [10,11] and especially in [11] where the two ways of forcing schemes investigated by [8] were also studied.

A recent numerical investigation [7] of decaying Burgers turbulence indicates that the effective number of degrees of freedom of the energy injection rate is provided by the ratio of the system size to the integral scale. Actually, a consistent result with [7] was previously obtained in [1] where the aspect ratio of the turbulent periodic box was changed from 1 to 8. In these cases, a large decrease is observed (by a factor 2) of the fluctuation rate of the

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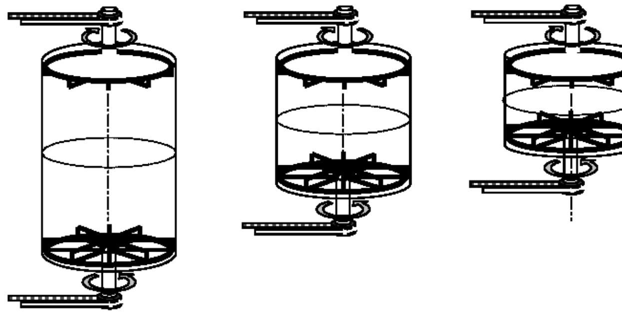

Fig. 1. The three experimental cells and their corresponding aspect ratio κ.

global kinetic energy. The decrease is consistent with an increase of the degrees of freedom.

In a different turbulent system, [6] measured the global heat flux in Rayleigh-Bénard convection. They found the effective number of degrees of freedom related to this global quantity to be given by the ratio of the system size to the thermal boundary thickness. This result is not in contradiction with [1] and [7] since the thermal boundary thickness is related to the large scale forcing of the turbulent flow. Our investigation is carried out with a strategy, similar to the two previous works [1,7]: we vary the aspect ratio of our experimental setup, and measure the effect on the fluctuations of the global injected power.

2 Experimental set-up

The turbulent flow is produced between two horizontal coaxial counter-rotating stirrers in a closed cylindrical cell filled with water. This set-up is also known as the Von Kármán swirl geometry. The stirrers are composed of Plexiglas disks of radius R = 8.75 cm, fitted with eight regularly disposed vertical blades. A detailed description of the forcing system and of its mechanical properties is available in reference [8]. The parameter that is modified in the present study is the aspect ratio κ, defined as the ratio D/H, where D is the diameter of the forcing stirrers and H is their distance apart. In our case the three different values of κ are achieved by using cylinders of same diameter but of smaller heights H, as shown in Figure 1. The geometric characteristics for each cell is given in table 1. The three values of the aspect ratio κ are 1, 1/2, 5/2. The total mass of water M_0 in a cell is different from the mass comprised between the stirrers because of a small gap filled with water between the disks and the cell sides. Experiments are carried out for the following counter rotating regime of rotation frequencies: $f_{rot} = \Omega_{rot}/2\pi = 4, 5, 6, 7$ and $8Hz$ with $\Omega_1 = -\Omega_2 = \Omega_{rot}$, are the angular velocities of both stirrers. The Reynolds number is defined for this experimental geometry by:

$$Re = \frac{\Omega_{rot} R^2}{\nu}. \quad (1)$$

According to this definition, the Reynolds number does not depend on the cell's height. The fluid being water for

Table 1. Geometric characteristics of the three experimental cells.

κ	H (cm)	D/H	M_0 (kg)	Mass between disks (kg)
1/2	32	0.547	10.95	9.07
1	19	0.92	7.26	5.39
5/2	7	2.5	3.58	1.98

all the experimental runs, the Reynolds number is directly proportional to the frequency of rotation and takes values comprised in the range 190000 to 380000. Time series of torques, $\Gamma_1(t)$ and $\Gamma_2(t)$ supplied by both servomotors driving each disks are recorded with a sampling frequency of $1kHz$. The data acquisition chain and the torque measurements technique are described lengthily in reference [8]. as well as the computation of the total power injected $P(t)$ in the turbulent flow. For each experimental run, statistical analysis is performed from the temporal series of $P(t)$.

3 Results and discussions

The global energy injection rate, $\epsilon(t)$ is obtained by dividing the total injected power P(t) by the total mass M_0 of the fluid in the experimental cell:

$$\epsilon(t) = \frac{P(t)}{M_0}. \quad (2)$$

We denote by $\delta\epsilon'$ the variable ϵ centered about its mean value and reduced by its root mean square rms value: $\delta\epsilon' = \frac{\epsilon - \langle\epsilon\rangle}{\delta\epsilon_{rms}}$, with $\delta\epsilon_{rms} = \sqrt{\langle\epsilon - \langle\epsilon\rangle\rangle^2}$ and $\langle ... \rangle$ denotes a temporal average. The probability density functions (PDFs) of ϵ are represented in Figure 2 for the 3 aspect ratio. At a given aspect ratio, we can see the self-similar behavior with the Reynolds number since all the normalized PDF's collapse on a single curve. We can observe a significant effect of the aspect ratio on the asymmetry of the PDFs compared to the Gaussian curve displayed on each graph. The asymmetry disappears for the largest aspect ratio, but reinforced for the aspect ratio of 1. However, the PDFs are never very asymmetric. A measurement of the asymmetry is given by the skewness which is defined as:

$$S = \frac{\langle\delta\epsilon'^3\rangle}{\delta\epsilon_{rms}^3}. \quad (3)$$

In our case, the larger asymmetry is obtained with a skewness of -0.2 (see Fig. 3a) which is much smaller than the asymmetry reported in previous works [1–3] who found a skewness inferior to -1. Small asymmetry PDFs comparable to our result have been recently reported in DNS [10] and [11]. For $\kappa = 5/2$ in Figure 2c, the PDF's become Gaussian. This essential ingredient is consistent with an averaging effect due to an increase of the number of degrees of freedom. We show the evolutions with Re and κ of both the mean energy injection rate $\langle\epsilon\rangle$ in Figure 3b and its fluctuations $\delta\epsilon_{rms}$ in Figure 3c. Both quantities are scaled by the Kolmogorov arguments [13]. We cannot distinguish any variations with the Reynolds number for both

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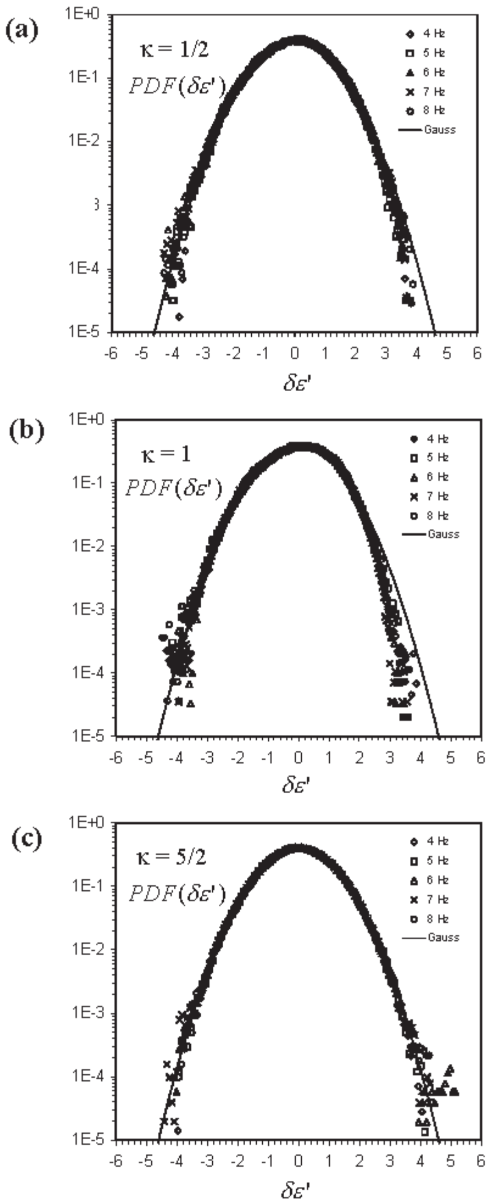

Fig. 2. The PDFs of the reduced (see text) energy injection rate $\delta\epsilon'$ for (a) the aspect ratio $\kappa = 1/2$, (b) $\kappa = 1$ and (c) $\kappa = 2.5$.

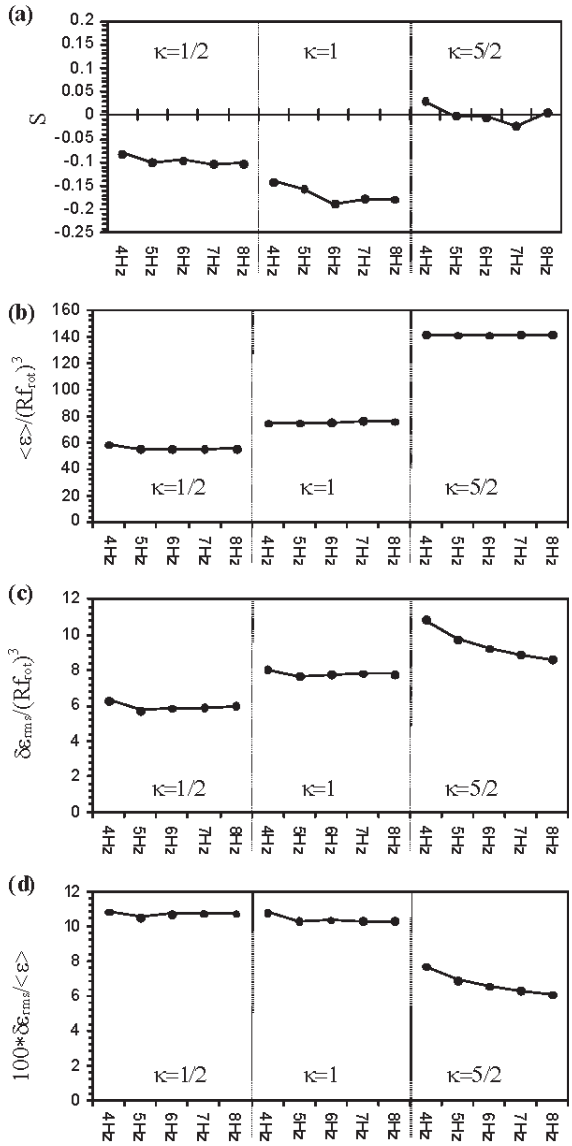

Fig. 3. Evolution with the counter rotating frequency f_{rot} and the aspect ratio of (a) the skewness, (b) the mean, (c) the fluctuation, (d) the rate of fluctuation of the energy injection rate. In (b) and (c) the energy injection rate is scaled by $(R * f_{rot})^3$.

the aspect ratio $\kappa = 1/2$ and $\kappa = 1$. This independence is in agreements with previous measurements of [8]. However, this is not as clear for $\delta\epsilon_{rms}$ at $\kappa = 5/2$, where a decrease is observed in Figure 3c. A reasonable explanation for this decrease can be found in the spectra of the energy injection rate displayed in Figure 4. Both series of spectra for both aspect ratio $\kappa = 1/2$ and $\kappa = 1$ behave similarly: the spectra extends further to the large frequencies as the rotation frequency increases (for frequencies smaller than 30 Hz). In contrast, the spectra of the $\kappa = 5/2$ series seem to be attenuated at a given frequency whatever the rotation frequency of the disks. Actually, our measurements technique has a high frequency cut-off [8] around 20 Hz (displayed as the cut-off in Fig. 4). It is then possible that the measurements at $\kappa = 5/2$ become subjected to this filtering and hence, the larger the rotation frequency is, the more damped the amplitudes of the fluctuations are. In this sense, we believe the measurements performed at the lower rotation frequency of $4Hz$ is probably the most reliable. When the aspect ratio is increased from $\kappa = 1$ to $\kappa = 5/2$ both the mean and the fluctuations of the energy injection rate are increased. A consequence of these variations is the reduction (by a factor of about 1.4 for 4 Hz) in Figure 3d of the fluctuation rate $\delta\epsilon_{rms}/\langle\epsilon\rangle$ for $\kappa = 5/2$. Since the fluctuation rate is smaller and the shape tends to a Gaussian, we have here an indication that the number of degrees of freedom in the system are increased when the aspect ratio goes from $\kappa = 1$ to $5/2$. The question is what could the degrees of freedom be related to? A possible answer is that the degrees of freedom could correspond to

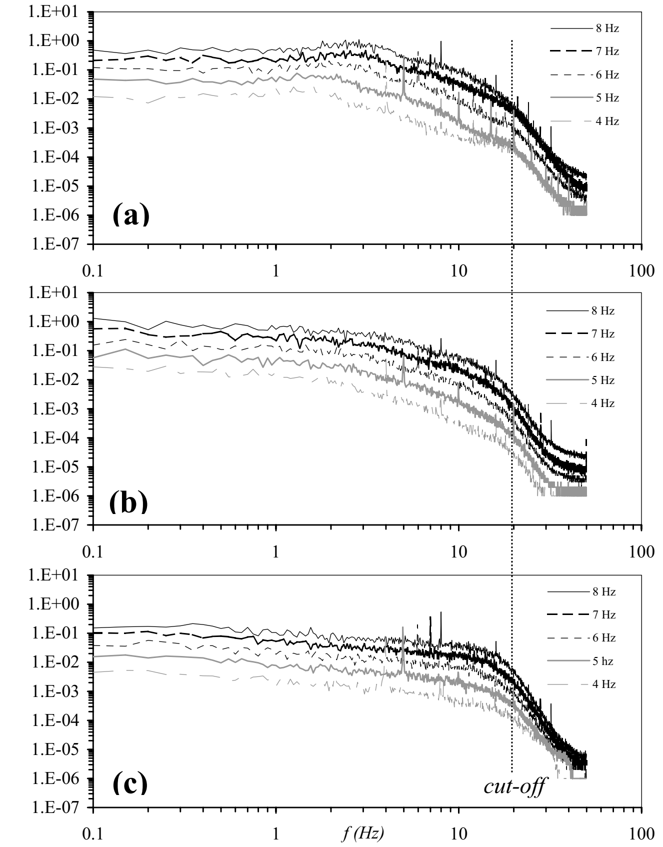

Fig. 4. Power spectra of the injected power for the three aspect ratio. (a) $\kappa = 1/2$; (b) $\kappa = 1$; (c) $\kappa = 5/2$.

some independent large scale structures whose number increases as the aspect ratio increases. In this type of flow, and for the counter rotating regime, it has been shown that the equatorial plan region corresponds to a circular shear in which large scale structures are formed through a Kelvin Helmholtz-like instability. The number of these structures depends crucially on the aspect ratio of the experiment. In the case of $\kappa = 1$, there are 2 structures [14] and for $\kappa = 2$ [15], there are 4 or 5 structures. We are then tempted to relate these structures to the degrees of freedom. From our measurements, the number of independent large scale structures can be estimated by two ways. The first consists in calculating an integral scale $L \propto \frac{(f_{rot}R)^3}{\langle\epsilon\rangle}$ from the mean energy injection rate and saying that $N \propto R/L$. The second consists in the decrease of the rate of fluctuation of the energy injection rate that should scale as $\frac{\epsilon_{rms}}{\langle\ \epsilon\rangle} \propto 1/\sqrt{N}$ resulting of the convolution of the statistics of N independent large scale structures. Both estimations are as follows:

$$N_1 = 2\frac{\langle\epsilon\rangle}{\langle\epsilon\rangle_{\kappa=1}} \quad or \quad N_2 = 2\frac{(\langle\epsilon\rangle/\epsilon_{rms})^2}{(\langle\epsilon\rangle/\epsilon_{rms})^2_{\kappa=1}}. \qquad (4)$$

Both are normalized in order to have N=2 for $\kappa = 1$ as observed in [14]. The computed number of structures are presented in table 2. For $\kappa = 1/2$ and $\kappa = 1$, N_1 and N_2 do not depend on the rotation frequency (i.e. $\langle\ \epsilon\rangle$ and ϵ_{rms} are constant). For $\kappa = 5/2$,we actually find the number of structures to increase as observed in [15], since we have $N_1 \sim 4$ and $4 < N_2 < 6$. For the aspect ratio of $\kappa = 1/2$, N_1 and N_2 seems to saturate around 2. Physically, we can argue that the structure size cannot exceed a length corresponding to the cell's diameter. The number of structures should then saturate as the aspect ratio goes to zero.

Table 2. Estimation of the evolution of the number of large scale structures from injection rate measurements as defined in equation (4).

κ	N_1	N_2	κ	N_1	N_2
1/2	1.48	1.95	5/2(6 Hz)	3.74	5.12
1	2	2	5/2(7 Hz)	3.74	5.56
5/2(4 Hz)	3.74	3.76	5/2(8 Hz)	3.74	5.9
5/2(5 Hz)	3.74	4.62			

4 Conclusion

It is possible to draw some basic ideas about the degrees of freedom of this confined turbulent flow. Similarly to the finding in [7] and [1], the number of degrees of freedom that are integrated over the stirring disks of radius R corresponds to the number of the large scale structure in the flow, which depends on the aspect ratio of the experiment. The larger the aspect ratio, the larger the number of elementary structures. In consequence the PDF shape tends to a Gaussian and the fluctuation rate is decreased as expected by the central limit theorem.

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